Substitution Property

WORKED Example

Evaluate each algebraic expression if $a = 2$ and $b = 3$.

a $a + 5b$ **b** $4ab$

THINK	WRITE
a 1 Replace a with 2 and b with 3 and insert a multiplication sign between 5 and b (as $5b = 5 \times b$).	**a** $a + 3b = 2 + 5 \times 3$
2 To evaluate, perform multiplication first, followed by addition.	$= 2 + 15$ $= 17$
b 1 Replace a with 2 and b with 3 and insert multiplication signs between the coefficient and the pronumerals.	**b** $4ab = 4 \times 2 \times 3$
2 To evaluate, multiply all numbers together.	$= 24$

Need more help with this? Click here for tutorials

Evaluate each using the values given.

1) $m + p^2 + p$; use $m = 6$, and $p = 5$

2) $3^2 + z - x$; use $x = 2$, and $z = -5$

3) $\frac{yz}{3} + y$; use $y = 3$, and $z = 3$

4) $p^2 + m + m$; use $m = 1$, and $p = -1$

5) $m - \frac{m}{4} - p$; use $m = 8$, and $p = 2$

6) $q \times \frac{p}{6} - p$; use $p = -6$, and $q = 5$

7) $\frac{x}{4} + y + x$; use $x = 4$, and $y = 3$

8) $xz \times \left(-\frac{4}{4}\right)$; use $x = 9$, and $z = 4$

9) $pr(q - p)$; use $p = -7$, $q = -9$, and $r = 5$

10) $\frac{y}{4} + 6 + z$; use $y = -4$, and $z = -1$

11) $(-5)(y - 4) - z$; use $y = 10$, and $z = 7$

12) $8 + q - p + 2$; use $p = 7$, and $q = -6$

13) $\frac{h}{6} - jk$; use $h = -6$, $j = 10$, and $k = 2$

14) $r + \frac{p}{5} - q$; use $p = -5$, $q = 7$, and $r = 9$

15) $(m - p)^2 - n$; use $m = -3$, $n = -1$, and $p = -6$

Solving Equations

Plan

How can you isolate the variable?
To isolate the variable, you have to remove the +4 from the left side of the equation.

Problem 1 Solving a One-Step Equation

What is the solution of $x + 4 = -12$?

Write the original equation.	$x + 4 = -12$
Use the Subtraction Property of Equality.	$x + 4 - 4 = -12 - 4$
Simplify.	$x = -16$
Check Substitute −16 for *x* in the original equation.	$-16 + 4 \stackrel{?}{=} -12$
The solution checks.	$-12 = -12$ ✔

Subtraction is the inverse operation of addition, so subtract 4 from each side.

Plan

How do you solve an equation with the variable on both sides?
Choose a side for the variable and remove it from the other side.

Problem 2 Solving a Multi-Step Equation

What is the solution of $-27 + 6y = 3(y - 3)$?

Write the original equation.	$-27 + 6y = 3(y - 3)$
Use the Distributive Property.	$-27 + 6y = 3y - 9$
Add 27 to each side.	$6y = 3y + 18$
Subtract 3*y* from each side.	$3y = 18$
Divide each side by 3.	$y = 6$

Need more help? Get it here.

Exercises

Solve each equation. See Problem 1.

10. $h - 12 = 6$ **11.** $-\frac{x}{3} = 27$ **12.** $4t = 48$

Solve each equation. Check your answer. See Problem 2.

13. $7w + 2 = 3w + 94$

To start, subtract 2 from each side. $7w + 2 - 2 = 3w + 94 - 2$

14. $15 - g = 23 - 2g$

15. $5y + 1.8 = 4y - 3.2$

16. $6a - 5 = 4a + 2$

17. $4y - 8 - 2y + 5 = 0$

18. $6(n - 4) = 3n$

19. $5(2 - g) = 0$

Solving Proportions

Example

About 15 of every 1000 light bulbs assembled at the Brite Lite Company are defective. If the Brite Lite Company assembles approximately 13,000 light bulbs each day, about how many are defective?

Set up a proportion to solve the problem. Let x represent the number of defective light bulbs per day.

$$\frac{15}{1000} = \frac{x}{13,000}$$

$15(13,000) = 1000x$	Cross Products Property
$195,000 = 1000x$	Simplify.
$\frac{195,000}{1000} = x$	Divide each side by 1000.
$195 = x$	Solve for the variable.

About 195 of the 13,000 light bulbs assembled each day are defective.

Find more help on solving proportions here.

Algebra **Use the Cross Products Property to solve each proportion.**

1. $\frac{3}{5} = \frac{x}{25}$
2. $\frac{x}{4} = \frac{9}{2}$
3. $\frac{x-2}{8} = \frac{3}{4}$
4. $\frac{y}{3} = \frac{y+6}{8}$
5. $\frac{x}{4} = \frac{77}{28}$
6. $\frac{3}{4y} = \frac{9}{138}$
7. $\frac{6}{d+5} = \frac{3}{d+1}$
8. $\frac{8}{2y-3} = \frac{6}{y+4}$
9. $\frac{x}{4} = \frac{13}{52}$
10. $\frac{x}{2x+1} = \frac{16}{40}$
11. $\frac{9}{10} = \frac{9x}{70}$
12. $\frac{2}{7} = \frac{b+1}{56}$
13. $\frac{11}{y} = \frac{9}{27}$
14. $\frac{3}{34} = \frac{m}{51}$
15. $\frac{(x+1)}{(x+1)} = \frac{10}{14}$
16. $\frac{7}{50} = \frac{x}{30}$

Simplifying Radicals

A radical expression is in simplest form when all of the following are true.

- The radicand has no perfect square factors other than 1.
- The radicand does not contain a fraction.
- A denominator does not contain a radical expression.

radical symbol ↓ $\sqrt{a}$ ← radicand

Example 1

Simplify the expressions $\sqrt{2} \cdot \sqrt{8}$ and $\sqrt{294} \div \sqrt{3}$.

$\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$ Write both numbers under one radical.

$= \sqrt{16}$ Simplify the expression under the radical.

$= 4$ Factor out perfect squares and simplify.

$$\sqrt{294} \div \sqrt{3} = \sqrt{\frac{294}{3}}$$
$$= \sqrt{98}$$
$$= \sqrt{49 \cdot 2}$$
$$= 7\sqrt{2}$$

Example 2

Write $\sqrt{\frac{4}{3}}$ in simplest form.

$\sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}}$ Rewrite the single radical as a quotient.

$= \frac{2}{\sqrt{3}}$ Simplify the numerator.

$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$ (a form of 1) to remove the radical from the denominator. This is called *rationalizing the denominator.*

$= \frac{2\sqrt{3}}{3}$

Some help on this here.

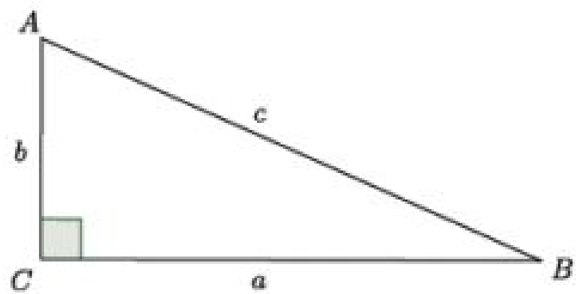
Exercises

Simplify each expression.

1. $\sqrt{5} \cdot \sqrt{10}$
2. $\sqrt{243}$
3. $\sqrt{128} \div \sqrt{2}$
4. $\sqrt{\frac{125}{4}}$
5. $\sqrt{6} \cdot \sqrt{8}$
6. $\frac{\sqrt{36}}{\sqrt{3}}$
7. $\frac{\sqrt{144}}{\sqrt{2}}$
8. $\sqrt{3} \cdot \sqrt{12}$
9. $\sqrt{72} \div \sqrt{2}$
10. $\sqrt{169}$
11. $28 \div \sqrt{8}$
12. $\sqrt{300} \div \sqrt{5}$
13. $\sqrt{12} \cdot \sqrt{2}$
14. $\frac{\sqrt{6} \cdot \sqrt{3}}{\sqrt{9}}$
15. $\frac{\sqrt{3} \cdot \sqrt{15}}{\sqrt{2}}$

Pythagorean's Theorem and Converse

Given a right triangle ABC with C being the vertex of the right angle, then the sides $\overline{AC}$ and $\overline{BC}$ are called the *legs* of $\triangle ABC$, and $\overline{AB}$ is called the *hypotenuse* of $\triangle ABC$.



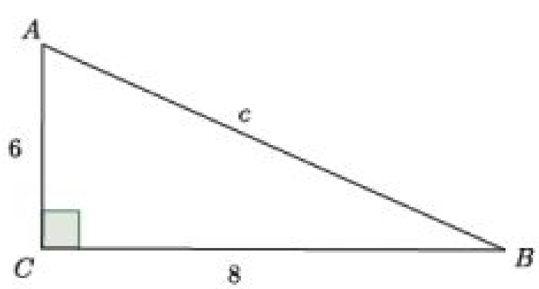
Take note of the fact that side a is opposite the angle A, side b is opposite the angle B, and side c is opposite the angle C.

The Pythagorean theorem states that for any right triangle, $a^2 + b^2 = c^2$.

Example 1

Now that we know what the Pythagorean theorem is, let's practice using it to find the length of a hypotenuse of a right triangle.

Determine the length of the hypotenuse of the right triangle.



The Pythagorean theorem states that for right triangles $a^2 + b^2 = c^2$, where a and b are the legs, and c is the hypotenuse. Then,

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2. \end{aligned}$$

Since we know that $100 = 10^2$, we can say that the hypotenuse c is 10.

The converse of the Pythagorean theorem states that if a triangle with side lengths a, b, and c satisfies $a^2 + b^2 = c^2$, then the triangle is a right triangle.

For more help on Pythagorean and Converse, click here.

Exercises

Determine whether the given lengths can be side lengths of a right triangle.

1. 15, 36, 39

2. 3, 7, 10

3. 8, 15, 17

4. $\sqrt{3}, \sqrt{4}, \sqrt{5}$

5. 6, 7, 8

6. 12, 16, 20

For the values given, *a* and *b* are legs of a right triangle. Find the length of the hypotenuse. If necessary, round to the nearest tenth.

7. $a = 6$, $b = 8$

8. $a = 5$, $b = 9$

9. $a = 4$, $b = 10$

10. $a = 9$, $b = 1$

11. $a = 7$, $b = 3.5$

12. $a = 1.4$, $b = 2.3$

Use the Pythagorean Theorem to answer each question.

13. A 20-ft ladder is placed 5 ft from the base of a building. How high on the building will the ladder reach?

14. A soccer field is 80 yd long and 35 yd wide. What is the diagonal distance across the field?

Basic Area

The formula to calculate the area of a parallelogram is $A = bh$, where b represents the base and h represents the height of the parallelogram.

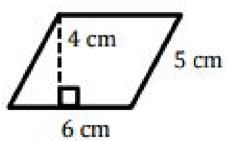
The height of a parallelogram is the line segment perpendicular to the base. The height is usually drawn from a vertex that is opposite the base.

Need help with area of parallelograms and triangles?

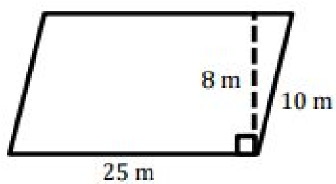
Exercises

1. Find the area of each parallelogram below. Note that the figures are not drawn to scale.

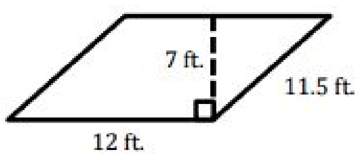
a.



b.



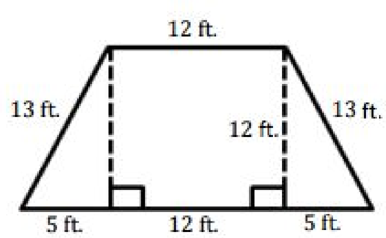
c.



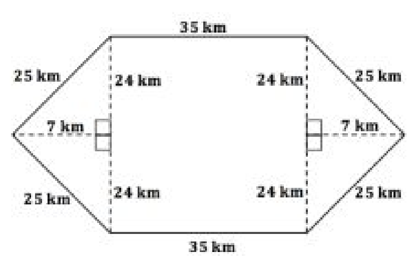
2. Can we use the formula $A = \frac{1}{2} \times \text{base} \times \text{height}$ to calculate the area of triangles that are not right triangles? Explain your thinking.

Calculate the area of each shape below. Figures are not drawn to scale.

3.



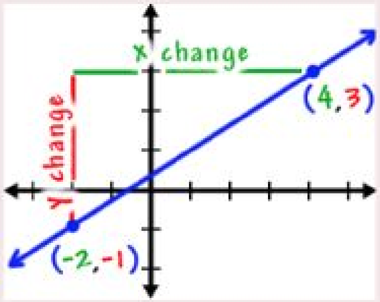
4.



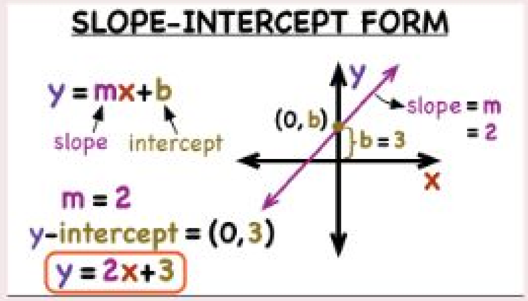
5. Immanuel is building a fence to make an enclosed play area for his dog. The enclosed area will be in the shape of a triangle with a base of 48 m. and an altitude of 32 m. How much space does the dog have to play?

Slope

<table>
<tr><th>Slope</th><th>Slope-Intercept Form</th></tr>
<tr><td>A measure of the steepness of a line. If (x_1, y_1) and (x_2, y_2) are any two points on the line, the slope of the lines, known as m, is represented by the equation $\frac{y_2 - y_1}{x_2 - x_1}$.</td><td>The slope-intercept form for a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.</td></tr>
<tr><td>



Rise
Run
</td><td>

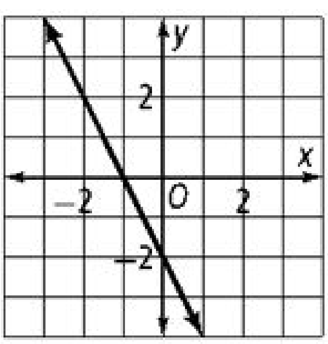


</td></tr>
</table>

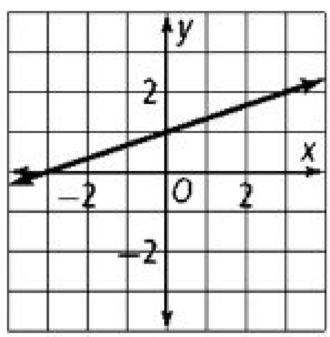
Some help on slope and equations of lines can be found [here](#).

Find the slope of each line.

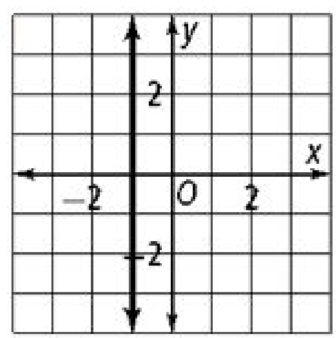
1.



2.



3.



Find the slope and y-intercept.

4. $y = 6x + 8$

5. $3x + 4y = -24$

6. $2y = 8$

7. $y = \frac{-3}{4}x - 8$

8. $2y = 3x - 1$

9. $4x - 5y = 2$

A line passes through the given points. Write an equation for the line in slope-intercept form.

10. (–2, 4) and (3, 9)

11. (1, 6) and (9, –4)

12. (0, –7) and (–1, 0)

13. (7, 0) and (3, –4)

14. (0, 0) and (–7, 1)

15. (10, 0) and (0, 7)