Welcome A.P. Calculus Students

Congratulations on your decision to enroll in A.P. Calculus. Electing to take an A.P. course may have been an arduous task; however, the intellectual stimulus and challenges that drive the curriculum will be a reward in and of itself, regardless of the outcome in May.

To make the transition from Pre-calculus to A.P. Calculus, our basic mathematics skills: graphing functions, factoring, and trigonometry must be top notch. You have an ample time-line (the summer) to accomplish this goal.

Your summer assignments are to complete the Cumulative Review, #1-100; and to be familiar with the 13 graphs of the parent functions. Both of these documents are attached to this letter, along with helpful suggestions from past A.P. Calculus students.

There will be a help session on Wednesday, August 8, at 3 p.m. in my classroom (3-216).

Be prepared for a quiz on Graphs of Parent Functions on DAY 1 (August 13).

TI-89 Graphing Calculators will be issued to students at the beginning of the school year.

Have a good summer. I am looking forward to experiencing A.P. Calculus with you next school year.

Sincerely,

Mr. Howery

Parent Functions

Constant: $f(x) = a$

Identity: $f(x) = x$

Absolute Value: $f(x) = |x|$

Reciprocal: $f(x) = 1/x$

Quadratic: $f(x) = x^2$

Power: $f(x) = x^n$ — n even / n odd

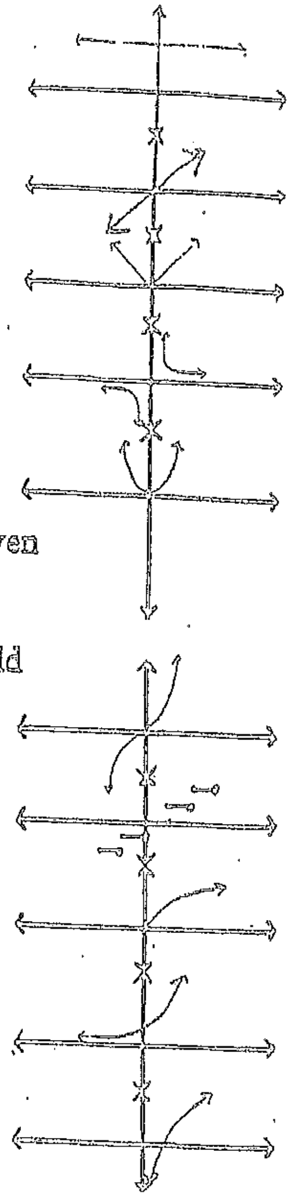
Cubic: $f(x) = x^3$

Greatest Integer: $f(x) = [x]$

Square Root: $f(x) = \sqrt{x}$

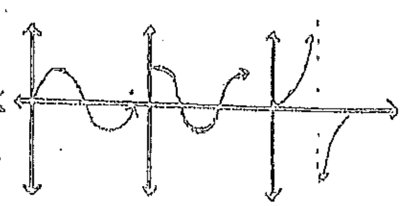
Exponential: $f(x) = a^x$

Logarithmic: $f(x) = \log_a x$



Trigonometric:

$f(x) = \sin x$
$f(x) = \cos x$
$f(x) = \tan x$



AP Calculus Summer Homework Name ____________________

Congratulations on your decision to enroll in A.P. Calculus. Electing to take an A.P. course may have been an arduous task; however, the intellectual stimulus and challenges that drive the curriculum will be a reward in and of itself, regardless of the outcome in May.

To make the transition from Pre-calculus to A.P. Calculus, our basic mathematics skills: graphing functions, factoring, and trigonometry must be top notch. You have an ample time-line (the summer) to accomplish this goal. I also want to emphasize the importance of understanding the prerequisite skills to succeed in Calculus. The MOST common errors I see throughout the year are not calculus mistakes, but algebra mistakes! This packet will highlight the skills that will serve as the foundation of Calculus throughout the year.

First, you should be able to factor polynomials to find zeroes of a function.
For example:
$3x^2 + 12x - 63 = 0$ can become $3(x^2 + 4x - 21) = 0$, which can be factored again to $3(x + 7)(x - 3) = 0$
Using the zero product property, x can equal 3 and -7. We do not use the quadratic formula in Calculus.
Also, you can only find the zeroes of a quadratic function if it equals zero.

1. $x^2 + 2x - 8 = 0$

4. $x^2 - 4x + 9 = 5$

2. $x^2 - 25 = 0$

5. $4x^2 - 8x = 0$

3. $3x^2 + 27x + 42 = 0$

6. $5x^2 + 30x + 45 = 0$

Factoring also helps us simplify fractions and rational functions (which are functions that have x in the denominator).

For example:
$f(x) = \frac{x^2+6x-7}{x-1}$ can be factored to $f(x) = \frac{(x+7)(x-1)}{x-1}$ which can simplify to $f(x) = x + 7$, a linear function.

7. $f(x) = \frac{x^2-x-6}{x-3}$

9. $f(x) = \frac{x^2+10x-24}{x^2-7x+10}$

8. $f(x) = \frac{x^2+5x+4}{x+4}$

10. $f(x) = \frac{x^2-9x+18}{x^2+x-42}$

You must be able to understand how to do basic operations with fractions.

Adding/Subtracting: $\frac{2}{5}+\frac{7}{8}$ ...you must find a common denominator, which you can do by just multiplying each number by the denominator of the other fraction. $\frac{2(8)}{5(8)}+\frac{7(5)}{8(5)}=\frac{16}{40}+\frac{35}{40}=\mathbf{\frac{51}{40}}$

Multiplying: $\frac{2}{5}\times\frac{7}{8}$ ...you just multiply the top, and multiply the bottom (do not cross multiply; only do that if you see "=" between fractions). Answer: $\frac{2\times7}{5\times8}=\frac{14}{40}=\mathbf{\frac{7}{20}}$

Dividing: $\frac{2}{5}\div\frac{7}{8}$ this can also be written as $\frac{\frac{2}{5}}{\frac{7}{8}}$...you just change the "÷" to "×" and take the reciprocal of the second fraction (keep-change-flip). Answer: $\frac{2}{5}\times\frac{8}{7}=\mathbf{\frac{16}{35}}$

11. $\frac{2}{3}+\frac{5}{2}$

12. $\frac{1}{4}-\frac{3}{8}$

13. $\frac{1}{2}-1$

14. $\frac{1}{x}+\frac{1}{y}$

15. $\frac{2}{3}\times\frac{5}{2}$

16. $\frac{1}{4}\div\frac{3}{8}$

17. $\frac{3}{2}\div 3$

18. $\frac{\frac{x}{x+y}}{x}$

Fractions also appear in exponents. The denominator becomes a radical, while the numerator remains as the exponent (remember that negative numbers have no roots that are even).

$25^{\frac{3}{2}}=\sqrt{25}^3$ you can either square root 25 first, or cube 25 first. It's easier to square root 25, and 5^3 = **125**

$8^{\frac{2}{3}}=\sqrt[3]{8^2}$ again, you may cube root 8 first (which is 2), or square 8 first. 2^2 = **4**

Exponents can also be negative, which turns the result into a reciprocal.

$25^{-\frac{1}{2}}=\frac{1}{\sqrt{25}}=\frac{1}{5}$ $\qquad$ $\frac{6}{27^{\frac{-2}{3}}}=6\left(27^{\frac{2}{3}}\right)=6\left(\sqrt[3]{27^2}\right)=6(3^2)=6(9)=\mathbf{54}$

19. $(-27)^{\frac{1}{3}}$

20. $16^{\frac{3}{2}}$

21. $(-8)^{\frac{5}{3}}$

22. $(-16)^{\frac{1}{2}}$

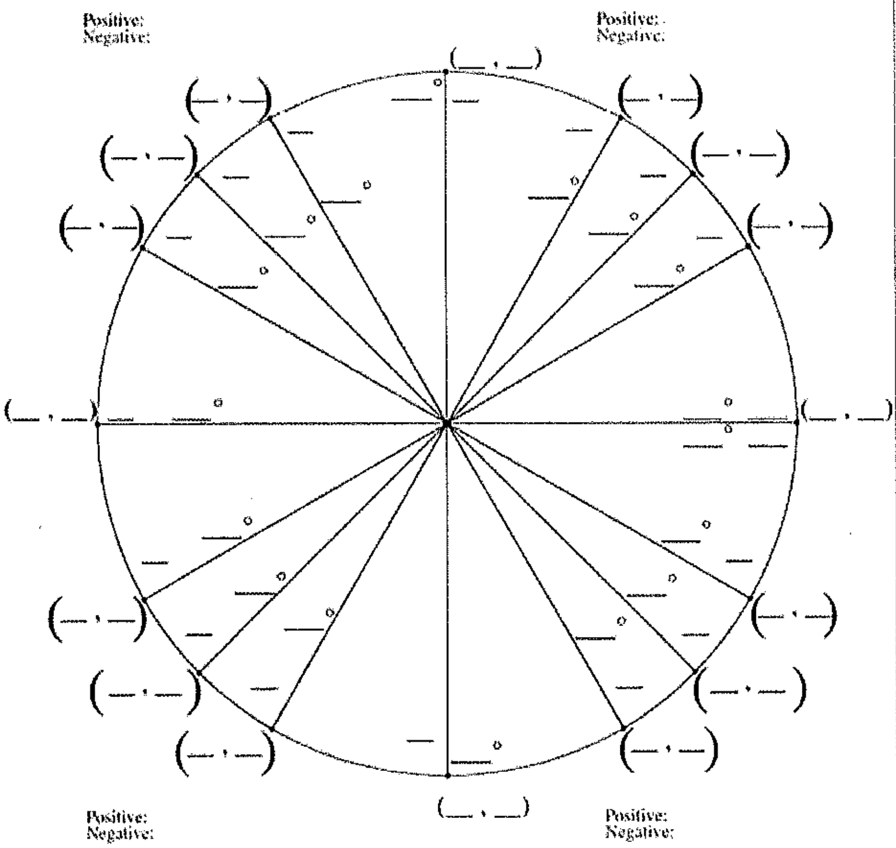
23. Simplify: $(\sqrt{x})^8$

24. Simplify: $x^{\frac{3}{2}}(x^{-\frac{5}{2}})$

Trigonometry appears in Calculus as well. You must be able to convert from radians to degrees and vice versa. To convert from degrees to radians, divide the degree measure by 180 and multiply it by pi.

For example: $60° = \frac{60}{180}\pi = \frac{\pi}{3}$ **radians**. For rad. to deg., do the opposite: $\frac{\pi}{2}$ radians $= \frac{1}{2}180° =$ **90°**
In the image below, fill in the angle measures (everything is in increments of 30°, 45°, and 60°). Fill in the radian measures as well. In the parentheses, write the coordinates (if you don't remember the coordinates, you can look them up).

Fill in The Unit Circle



When solving trig problems, you need to know what each function is referring to.

$sin\theta$ = the y coordinate of the angle	$csc\theta = \frac{1}{sin\theta}$	rule: $sin^2\theta + cos^2\theta = 1$
$cos\theta$ = the x coordinate of the angle	$sec\theta = \frac{1}{cos\theta}$	the above equation is the same as:
$tan\theta = \frac{sin\theta}{cos\theta}$	$cot\theta = \frac{cos\theta}{sin\theta}$	$(sin\theta)^2 + (cos\theta)^2 = 1$

These are some of the important trig problems to know. Some have already been answered.

25. $sin0 =$ **0**

26. $sin\frac{\pi}{6}$

27. $sin\frac{\pi}{2}$

28. $sin\pi$

29. $cos0$

30. $cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

31. $cos\frac{\pi}{6}$

32. $cos\pi$

33. $tan0$

34. $tan\frac{\pi}{4}$

35. $cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

36. $sin^{-1}(\frac{\sqrt{3}}{2})$

Logarithms show up in Calculus, but mostly with base *e*. The number *e* is approximately 2.718, and it occurs naturally in the universe, usually applying to exponential growth.

Recall that something like "$\log_2 32$ = x" means, "2 with what exponent equals 32?" or, $2^x = 32$. **x = 5.** The base in this example is 2. When the base is *e*, instead of writing "$\log_e 32$" we write Ln32.

Thinking back to the section on exponents, let's consider the function y = $\log_4 x$. This is basically saying $4^y = x$. No matter what value you input for y, the value for x will never be negative. In fact, it could never be zero either. Therefore, the domain of this function is x > 0, and you would say that $\log_4 x$ has no solution if x = 0 or x is any negative number. You could, however, input any value you wanted into y. Therefore, the range of this function is all real numbers, written as $-\infty < y < \infty$

Some other things to remember:
Ln1 = 0, because $e^0 = 1$ (anything to the 0 power is 1)
Ln*e* = 1, because $e^1 = e$
Lne^4 = 4, because when an exponent appears in a logarithm, you can bring it to the front, and 4ln*e* = 4(1)

State whether the following is true or false.

37. $\ln\sqrt{e} = \frac{1}{2}$

38. If y = $\ln(x^2 - 4)$, then 2 could be a possible value of x.

39. $e^{\ln 5} = 5$

40. There exists a real value for x, such that the expression $\ln(x^2 + 1)$ has no solution

41. The domain of ln|x| (Ln of the absolute value of x) is all real numbers except 0, written $x \neq 0$

42. lnx > 1 only if x > *e*

43. 0 < lnx < 1 only if 1 < x < *e*

44. lnx < 0 only if 0 < x < 1

Composite functions are functions that are inside of other functions. For example, $f(x) = (x^4 + 2x - 1)^3$ and $g(x) = \sin(2x)$ are both composite functions. In f(x), $x^4 + 2x - 1$ is the inside function, and in g(x), 2x is the inside function.

45. Which of the following are composite functions?

a. $\ln(x^2)$

b. $4(\frac{x+2}{x-6})^2$

c. $\frac{x^2+1}{x-1}$

d. $\cos(\sin(x))$

e. e^{5x}

f. $(x + 6)(x - 4)$

Odd functions are functions that, if you change the sign of x, the sign of y also changes: $f(-x) = -f(x)$
Even functions are functions that, if you change the sign of x, the y value is exactly the same: $f(-x) = f(x)$

For example, $f(x) = 3x^2$ is an even function because if you input 3 or -3, the output is still 27. However, $f(x) = x^3$ is an odd function because if you input 3, the output is 27, but if you input -3, the output is -27.

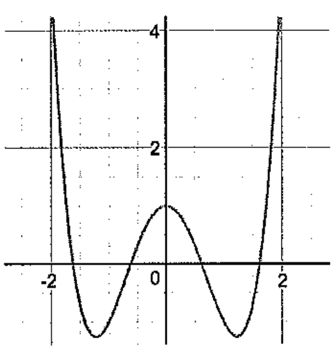
Determine whether the following functions are odd, even, or neither.

46. $f(x) = x^4 + 1$

47. $f(x) = x - 3$

48. $f(x) = \frac{3}{x^2-2}$

49.

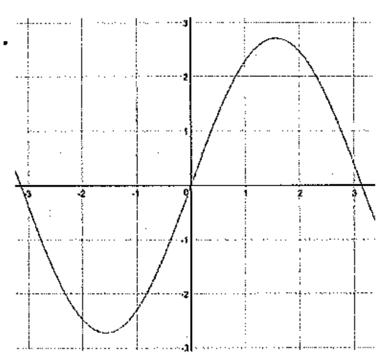


50. $f(x) = \sin x$

51. $f(x) = \cos x$

52. $f(x) = x^5 + x^3 - x$

53.



54. What is the symmetry of the graph of an even function? What about an odd function?

Some of the most common errors occur in problems that simply ask you to solve for x.
Solve the following equations for x. If other variables are present, x is not going to be a single value.

55. $4z + 10yx = 0$

62. $\frac{2x-1}{(x+2)(x^2+3)} = 0$

56. $\frac{x^2-16}{x^3} = 0$

63. $\frac{x^2+1}{(x-3)(x^2-7)} = 0$

57. $x^2 - 3x - 4 = 14$

64. $2y - 2zx = z + xy$

58. $y^2 + 3xy - 8x - 4y = 0$

65. $\frac{cosx}{x^2} = 0$

59. $(x - 5)^2 = 9$

66. $\frac{x+3}{6} = \frac{2x-6}{3}$

60. $12x^2 = 3x$

67. $4x - 2(x+1) = -8$

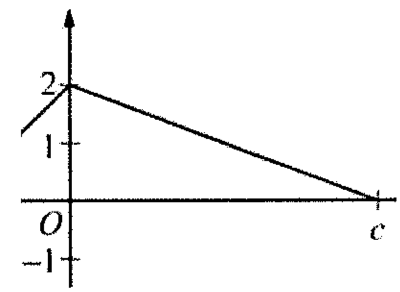
61. $\frac{8+x}{x} - 4 = 1$

68. $e^{2x}(x - 6) = 0$

Other common errors occur in problems that require you to create equations from context. Write, but do not solve, an equation or expression that could answer the following situations. The only problem I would like you to solve is number 75.

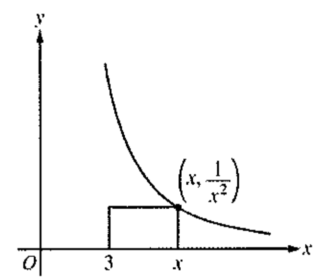
69. An isosceles right triangle has the same area as a quarter circle with radius 3. Find the length of the legs of the triangle. (don't actually find the length, just write an equation that would help you)

70. If the length of the segment that goes from x = 0 to x = c is 2.5, find c.



71. Find the slope of the segment to the right, in terms of c.

72. The points (3,0), (x,0), $(3,\frac{1}{x^2})$, and $(x,\frac{1}{x^2})$ form a rectangle. This rectangle and the equation $y = \frac{1}{x^2}$ are shown to the right. In terms of x, what is the area of the rectangle?



73. Write an expression that represents the sum of a number's reciprocal and five times its square.

74. A window is the shape of a square with a semicircle (half circle) on top. If the perimeter is 24, what is the radius of the semicircle?

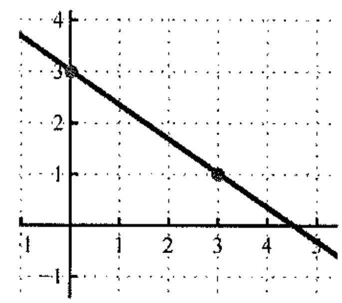
75. Find two numbers whose product is 363, and the sum of one number and three times the second is 66 (solve this using equations, don't guess and check).

76. In terms of x, what is the distance from any point on the function $y = 4 - x^2$ to the point (2,1)? The distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

You must have a complete understanding of linear functions. This includes the following topics:

a. Find the equation of a linear function from a graph
b. Find the equation of a linear function from a point and slope
c. Find the slope of a linear function from a graph
d. Find the slope of a linear function from two points
e. Find the y- and x- intercept of a linear function from a graph
f. Find the y- and x- intercept of a linear function from an equation
g. Know the slope formula ($\frac{y_2-y_1}{x_2-x_1}$), the slope-intercept equation (y = mx + b), the point-slope equation ($y - y_1 = m(x - x_1)$), and the standard form equation of a line (ax + by = c)
h. Know that parallel lines have the same slope; perpendicular lines have opposite reciprocal slopes
i. The equation of a vertical line is x = a, because it crosses the x-axis at point a. The slope is undefined
j. The equation of a horizontal line is y = a, because it crosses the y-axis at point a. The slope is zero.

77. What is the slope of the line in the graph pictured to the right?



78. What is the y-intercept of the graph pictured to the right?

79. What is the equation in slope-intercept form of the graph pictured to the right?

80. What is the equation in slope-intercept form of the line with slope of ¾ passing through point (8,-2)?

81. What is the equation in standard form of the line with slope 2/5 passing through point (-5,7)? (standard form typically does not have fractions, so you'll have to find a way to change the equation)

82. What is the slope, y-intercept, and x-intercept of the function 3x – 7y = 21?

83. The line parallel to y = 3x – 4 passes through point (1,6). What is the equation of that line?

84. The line perpendicular to 2x + 6y = 11 passes through point (-1,-1). What is the equation of that line?

85. What is the equation of the line that passes through points (-2,3) and (6,7)?

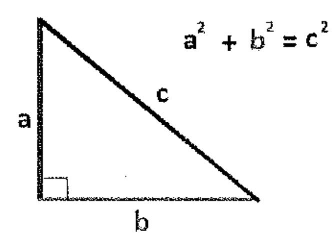
86. What is the average rate of change (this means use the slope formula) of the function $y = x^2 + 2x - 1$ between the points located at x = 3 and x = 5?

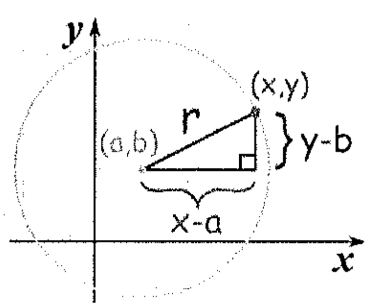
87. Find the equation of a vertical line that passes through point (1,-5).

88. Find the equation of a horizontal line that passes though point (3,2).

89. Find the equation of a line with y-intercept 3 and x-intercept 5.

The equation of a circle is derived from the Pythagorean Theorem, which has to do with the side lengths of right triangles. See the image to the right for the formula for the Pythagorean Theorem.
A circle can be made by taking the hypotenuse of a right triangle, which is length c, and spinning it around a point. See the image below. The equation of the circle is $(x-a)^2 + (y-b)^2 = r^2$, where point (a,b) is the center, and r is the radius.





90. What is the equation of a circle with center (1,-3) and radius 6?

91. What is the maximum and minimum coordinates of a circle with the equation $(x+4)^2 + (y-1)^2 = 81$?

)2. What is the area of the part of the circle above the x-axis with the equation $(x-7)^2 + y^2 = 16$?

Lastly, answer these random questions about functions.

What does it mean if:

93. a function is located in the first or second quadrant

94. f(2) = 5

95. a graph is a function

96. y is directly proportional to x

97. y is indirectly proportional to x

98. a function is a polynomial

99. a function has a horizontal asymptote at y = 2

100. a function has a vertical asymptote at x = 1

When you are finished with this summer packet, or if you need any help or explanations, email me at dhowery@pasco.k12.fl.us with questions. I will be available to offer help on Wednesday, August 8, from 3 – 4 pm, in room 3-216. This guide is by no means every prerequisite skill for Calculus, but if you understand all of the concepts, you are in great shape! If you would like to learn a little bit about Calculus before the school year starts, feel free to watch Khan Academy videos about any of the following topics:

- Limits, Continuity, and Asymptotes
- Derivatives and Differentiation
- Integrals and the Fundamental Theorems of Calculus

Have a great summer!
Mr. Howery

Suggestions from previous AP Calculus students:

1. Keep up!

2. Do all the homework.

3. Intellectual integrity is essential.

4. Learn how to use your graphing calculator.

5. Take good notes.

6. Limit absences and SRA's.

7. Don't give up.

8. You must have an excellent background in Algebra, Geometry, and Precalculus. Topics to review are:
   a. Trigonometric functions
      Graphs
      Unit circle
      Trigonometric identities

   b. Factoring and expanding

   c. Solving equations

   d. Writing equations of lines

   e. Properties of logarithms

   f. Exponential functions

   g. Graphs/behavior of basic functions

   h. Parametric functions (BC only)

   i. Polar functions (BC only)

   j. Sequences and Series (BC only)

9. Look to make connections. What's given? What do I know? What am I trying to find? What do I need?