Adding and Subtracting Fractions

You can add and subtract fractions when they have the same denominator. Fractions with the same denominator are called like fractions.

Example 1

a. Add $\frac{4}{5} + \frac{3}{5}$.

$\frac{4}{5} + \frac{3}{5} = \frac{4+3}{5} = \frac{7}{5} = 1\frac{2}{5}$ ← Add or subtract the numerators and keep the same denominator. →

b. Subtract $\frac{5}{9} - \frac{2}{9}$.

$\frac{5}{9} - \frac{2}{9} = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}$

Fractions with unlike denominators are called unlike fractions. To add or subtract unlike fractions, find the least common denominator (LCD) and write equivalent fractions with the same denominator. Then add or subtract the like fractions.

Example 2

Add $\frac{3}{4} + \frac{5}{6}$.

$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12}$ — Find the LCD. The LCD is the least common multiple (LCM) of the denominators. The LCD of 4 and 6 is 12. Write equivalent fractions.

$= \frac{9+10}{12} = \frac{19}{12}$, or $1\frac{7}{12}$ — Add like fractions and simplify.

To add or subtract mixed numbers, add or subtract the fractions. Then add or subtract the whole numbers. Sometimes when subtracting mixed numbers you have to regroup so that you can subtract the fractions.

Example 3

Subtract $5\frac{1}{4} - 3\frac{2}{3}$.

$5\frac{1}{4} - 3\frac{2}{3} = 5\frac{3}{12} - 3\frac{8}{12}$ — Write equivalent fractions with the same denominator.

$= 4\frac{15}{12} - 3\frac{8}{12}$ — Write $5\frac{3}{12}$ as $4\frac{15}{12}$ so you can subtract the fractions.

$= 1\frac{7}{12}$ — Subtract the fractions. Then subtract the whole numbers.

Still confused? Get some help [here].

Exercises

Add or subtract. Write each answer in simplest form.

1. $\frac{2}{7} + \frac{3}{7}$	**2.** $\frac{3}{8} + \frac{7}{8}$	**3.** $\frac{6}{5} + \frac{9}{5}$	**4.** $\frac{4}{9} + \frac{8}{9}$	**5.** $6\frac{2}{3} + 3\frac{4}{5}$
6. $1\frac{4}{7} + 2\frac{3}{14}$	**7.** $4\frac{5}{6} + 1\frac{7}{18}$	**8.** $2\frac{4}{5} + 3\frac{6}{7}$	**9.** $4\frac{2}{3} + 1\frac{6}{11}$	**10.** $3\frac{7}{9} + 5\frac{4}{11}$
11. $8 + 1\frac{2}{3}$	**12.** $8\frac{1}{5} + 3\frac{3}{4}$	**13.** $11\frac{3}{8} + 2\frac{1}{16}$	**14.** $\frac{7}{8} - \frac{3}{8}$	**15.** $\frac{9}{10} - \frac{3}{10}$
16. $\frac{17}{5} - \frac{2}{5}$	**17.** $\frac{11}{7} - \frac{2}{7}$	**18.** $\frac{5}{11} - \frac{4}{11}$	**19.** $8\frac{5}{8} - 6\frac{1}{4}$	**20.** $3\frac{2}{3} - 1\frac{8}{9}$

Multiplying and Dividing Fractions

To multiply two or more fractions, multiply the numerators, multiply the denominators, and simplify the product, if necessary.

Example 1

Multiply $\frac{3}{7} \cdot \frac{5}{6}$.

Method 1 Multiply the numerators and the denominators. Then simplify.

$$\frac{3}{7} \cdot \frac{5}{6} = \frac{3 \cdot 5}{7 \cdot 6} = \frac{15}{42} = \frac{15 \div 3}{42 \div 3} = \frac{5}{14}$$

Method 2 Simplify before multiplying.

$$\frac{\overset{1}{\cancel{3}}}{7} \cdot \frac{5}{\cancel{6}_{2}} = \frac{1 \cdot 5}{7 \cdot 2} = \frac{5}{14}$$

To multiply mixed numbers, change the mixed numbers to improper fractions and multiply the fractions. Write the product as a mixed number.

Example 2

Multiply $2\frac{4}{5} \cdot 1\frac{2}{3}$.

$$2\frac{4}{5} \cdot 1\frac{2}{3} = \frac{\overset{1}{\cancel{14}}}{5} \cdot \frac{5^{1}}{\cancel{3}} = \frac{14}{3} = 4\frac{2}{3}$$

To divide fractions, change the division problem to a multiplication problem. Remember that $8 \div \frac{1}{4}$ is the same as $8 \cdot 4$. To divide mixed numbers, change the mixed numbers to improper fractions and divide the fractions.

Example 3

a. Divide $\frac{4}{5} \div \frac{3}{7}$.

$\frac{4}{5} \div \frac{3}{7} = \frac{4}{5} \cdot \frac{7}{3}$ Multiply by the reciprocal of the divisor.

$= \frac{28}{15}$ Simplify.

$= 1\frac{13}{15}$ Write as a mixed number.

b. Divide $4\frac{2}{3} \div 7\frac{3}{5}$.

$4\frac{2}{3} \div 7\frac{3}{5} = \frac{14}{3} \div \frac{38}{5}$ Change to improper fractions.

$= \frac{\cancel{14}^{7}}{3} \cdot \frac{5}{\cancel{38}_{19}}$ Simplify.

$= \frac{35}{57}$ Multiply.

Still confused? Get some help [here]() for multiplying and [here]() for dividing.

Exercises

Multiply or divide. Write your answers in simplest form.

1. $\frac{2}{5} \cdot \frac{3}{4}$	**2.** $\frac{3}{7} \cdot \frac{4}{3}$	**3.** $1\frac{1}{2} \cdot 5\frac{3}{4}$	**4.** $3\frac{4}{5} \cdot 10$	**5.** $5\frac{1}{4} \cdot \frac{2}{3}$
6. $4\frac{1}{2} \cdot 7\frac{1}{2}$	**7.** $3\frac{2}{3} \cdot 6\frac{9}{10}$	**8.** $6\frac{1}{2} \cdot 7\frac{2}{3}$	**9.** $2\frac{2}{5} \cdot 1\frac{1}{6}$	**10.** $4\frac{1}{9} \cdot 3\frac{3}{8}$
11. $\frac{3}{5} \div \frac{1}{2}$	**12.** $\frac{4}{5} \div \frac{9}{10}$	**13.** $2\frac{1}{2} \div 3\frac{1}{2}$	**14.** $1\frac{4}{5} \div 2\frac{1}{2}$	**15.** $3\frac{1}{6} \div 1\frac{3}{4}$

Solving Equations

Plan

How can you isolate the variable?
To isolate the variable, you have to remove the +4 from the left side of the equation.

Problem 1 Solving a One-Step Equation

What is the solution of $x + 4 = -12$?

Write the original equation.	$x + 4 = -12$
Use the Subtraction Property of Equality.	$x + 4 - 4 = -12 - 4$
Simplify.	$x = -16$
Check Substitute −16 for x in the original equation.	$-16 + 4 \stackrel{?}{=} -12$
The solution checks.	$-12 = -12$ ✔

Subtraction is the inverse operation of addition, so subtract 4 from each side.

Plan

How do you solve an equation with the variable on both sides?
Choose a side for the variable and remove it from the other side.

Problem 2 Solving a Multi-Step Equation

What is the solution of $-27 + 6y = 3(y - 3)$?

Write the original equation.	$-27 + 6y = 3(y - 3)$
Use the Distributive Property.	$-27 + 6y = 3y - 9$
Add 27 to each side.	$6y = 3y + 18$
Subtract $3y$ from each side.	$3y = 18$
Divide each side by 3.	$y = 6$

Need more help? Get it here.

Exercises

Solve each equation. See Problem 1.

10. $h - 12 = 6$ **11.** $-\frac{x}{3} = 27$ **12.** $4t = 48$

Solve each equation. Check your answer. See Problem 2.

13. $7w + 2 = 3w + 94$

To start, subtract 2 from each side. $7w + 2 - 2 = 3w + 94 - 2$

14. $15 - g = 23 - 2g$

15. $5y + 1.8 = 4y - 3.2$

16. $6a - 5 = 4a + 2$

17. $4y - 8 - 2y + 5 = 0$

18. $6(n - 4) = 3n$

19. $5(2 - g) = 0$

Solving Systems Algebraically

Problem 1 Solving by Substitution

What is the solution of the system of equations? $\begin{cases} 3x + 4y = 12 \\ 2x + y = 10 \end{cases}$

Think

Which variable should you solve for first?
In the second equation, the coefficient of y is 1. It is the easiest variable to isolate.

Step 1
Solve one equation for one of the variables.

$$\begin{aligned} 2x + y &= 10 \\ y &= -2x + 10 \end{aligned}$$

Step 2
Substitute the expression for y in the other equation. Solve for x.

$$\begin{aligned} 3x + 4y &= 12 \\ 3x + 4(-2x + 10) &= 12 \\ 3x - 8x + 40 &= 12 \\ -5x + 40 &= 12 \\ x &= 5.6 \end{aligned}$$

Step 3
Substitute the value for x into one of the original equations. Solve for y.

$$\begin{aligned} 2x + y &= 10 \\ 2(5.6) + y &= 10 \\ 11.2 + y &= 10 \\ y &= -1.2 \end{aligned}$$

The solution is $(5.6, -1.2)$.

Problem 3 Solving by Elimination

What is the solution of the system of equations? $\begin{cases} 4x + 2y = 9 \\ -4x + 3y = 16 \end{cases}$

Think

How can you use the Addition Property of Equality?
Since $-4x + 3y$ is equal to 16, you can add the same value to each side of $4x + 2y = 9$.

$$\begin{aligned} 4x + 2y &= 9 \\ -4x + 3y &= 16 \end{aligned}$$

One equation has $4x$, and the other has $-4x$. You can add to eliminate the variable x.

Add. $5y = 25$

Solve for y. $y = 5$

Choose one of the original equations. $4x + 2y = 9$

Substitute for y. $4x + 2(5) = 9$

Solve for x. $4x = -1$

$x = -\frac{1}{4}$

The solution is $\left(-\frac{1}{4}, 5\right)$.

Problem 4 Solving an Equivalent System

What is the solution of the system of equations? $\begin{cases} ① \; 2x + 7y = 4 \\ ② \; 3x + 5y = -5 \end{cases}$

Think	Write
Multiply ① by 3 and ② by −2. This makes the x-terms opposites, and you can eliminate them.	① $2x + 7y = 4$ ③ $6x + 21y = 12$ ② $3x + 5y = -5$ ④ $-6x - 10y = 10$
Add ③ and ④. Solve for y.	$11y = 22$ $y = 2$
Now that you know the value of y, use either equation to find x.	① $2x + 7(2) = 4$ $2x + 14 = 4$ $2x = -10$ $x = -5$ The solution is $(-5, 2)$.

Think

How are the two equations in this system related?
Multiplying both sides of the first equation by -1 results in the second equation.

Problem 5 Solving Systems Without Unique Solutions

What are the solutions of the following systems? Explain.

A $\begin{cases} -3x + y = -5 \\ 3x - y = 5 \end{cases}$

$$0 = 0$$

Elimination gives an equation that is always true. The two equations represent the same line. This system has infinitely many solutions.

B $\begin{cases} 4x - 6y = 6 \\ -4x + 6y = 10 \end{cases}$

$$0 = 16$$

Elimination gives an equation that is always false. The two equations represent parallel lines. This system has no solution.

Need more help? Find it here.

Exercises

Solve each system by substitution. Check your answers. See Problem 1.

4. $\begin{cases} 4x + 2y = 7 \\ y = 5x \end{cases}$

To start, use the second equation to substitute for y in the first equation.

$$4x + 2y = 7$$
$$4x + 2(5x) = 7$$

5. $\begin{cases} 3c + 2d = 2 \\ d = 4 \end{cases}$

6. $\begin{cases} 4p + 2q = 8 \\ q = 2p + 1 \end{cases}$

7. $\begin{cases} x + 3y = 7 \\ 2x - 4y = 24 \end{cases}$

8. $\begin{cases} x + 6y = 2 \\ 5x + 4y = 36 \end{cases}$

9. $\begin{cases} y = 2x - 1 \\ 3x - y = -1 \end{cases}$

10. $\begin{cases} r + s = -12 \\ 2r - 3s = 6 \end{cases}$

Solve each system by elimination. See Problem 3.

7. $\begin{cases} x + y = 12 \\ x - y = 2 \end{cases}$

To start, add to eliminate the variable y.

$$\begin{array}{r} x + y = 12 \\ \underline{x - y = 2} \\ 2x \quad = 14 \end{array}$$

8. $\begin{cases} x + 2y = 10 \\ x + y = 6 \end{cases}$

9. $\begin{cases} 3a + 4b = 9 \\ -3a - 2b = -3 \end{cases}$

10. $\begin{cases} 4x + 2y = 4 \\ 6x + 2y = 8 \end{cases}$

11. $\begin{cases} 3u + 3v = 15 \\ -2u + 3v = -5 \end{cases}$

12. $\begin{cases} 3x + 2y = 6 \\ 3x + 3 = y \end{cases}$

13. $\begin{cases} 5x - y = 4 \\ 2x - y = 1 \end{cases}$

Solve each system by elimination. See Problems 4 and 5.

14. $\begin{cases} 4x - 6y = -26 \\ -2x + 3y = 13 \end{cases}$

To start, write an equivalent system with additive inverses by multiplying the second equation by 2.

$$\begin{cases} 4x - 6y = -26 \\ -4x + 6y = 26 \end{cases}$$

15. $\begin{cases} 9a - 3d = 3 \\ -3a + d = -1 \end{cases}$

16. $\begin{cases} 2a + 3b = 12 \\ 5a - b = 13 \end{cases}$

17. $\begin{cases} 2x - 3y = 6 \\ 6x - 9y = 9 \end{cases}$

18. $\begin{cases} 20x + 5y = 120 \\ 10x + 7.5y = 80 \end{cases}$

19. $\begin{cases} 6x - 2y = 11 \\ -9x + 3y = 16 \end{cases}$

20. $\begin{cases} 2x - 3y = -1 \\ 3x + 4y = 8 \end{cases}$

Solving Quadratic Equations

take note

Key Concept The Quadratic Formula

To solve the quadratic equation $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Some help on this here.

Plan

Should you write the equation in standard form?
Yes; write the equation in standard form to identify a, b, and c.

Think

Why is there only one solution?
When you add or subtract zero, you get the same number.

Hint

The Quadratic Formula works even when you can solve by factoring.

Problem 1 Using the Quadratic Formula

What are the solutions of each equation? Use the Quadratic Formula.

A $2x^2 - x = 4$

Write the original equation.	$2x^2 - x = 4$
Write in standard form.	$2x^2 - x - 4 = 0$
Find the values of a, b, and c.	$a = 2, b = -1, c = -4$
Write the Quadratic Formula.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Substitute for a, b, and c.	$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-4)}}{2(2)}$
Simplify.	$= \frac{1 \pm \sqrt{33}}{4}$
Write the solution.	$= \frac{1 + \sqrt{33}}{4}$ or $\frac{1 - \sqrt{33}}{4}$

B $x^2 + 6x + 9 = 0$

Write the original equation.	$x^2 + 6x + 9 = 0$
Find the values of a, b, and c.	$a = 1, b = 6, c = 9$
Substitute into $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.	$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$
Simplify.	$= \frac{-6 \pm \sqrt{36 - 36}}{2}$
Simplify under the radical.	$= \frac{-6 \pm \sqrt{0}}{2}$
Write the solution.	$= -3$

Check Solve by factoring.

Write the original equation.	$x^2 + 6x + 9 = 0$
Factor the perfect square trinomial.	$(x + 3)^2 = 0$
Find square roots.	$x + 3 = 0$
Solve for x.	$x = -3$

Think

What do you know about the factors of $x^2 - bx + c$?

The product of their constant terms is c.

The sum is $-b$.

Problem 1 Solving a Quadratic Equation by Factoring

What are the solutions of the quadratic equation $x^2 - 5x + 6 = 0$?

Write the original equation.	$x^2 - 5x + 6 = 0$
Factor the quadratic expression.	$(x - 2)(x - 3) = 0$
Use the Zero-Product Property.	$x - 2 = 0$ or $x - 3 = 0$
Solve for x.	$x = 2$ or $x = 3$

The solutions are $x = 2$ and $x = 3$.

Help on solving by factoring here.

Exercises

Solve each equation using the Quadratic Formula. See Problem 1.

11. $x^2 - 4x + 3 = 0$

To start, find the values of a, b, and c. $a = 1, b = -4, c = 3$

Substitute in the Quadratic Formula. $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$

12. $x^2 + 8x + 12 = 0$ **13.** $2x^2 + 5x = 7$ **14.** $3x^2 + 2x - 1 = 0$

15. $x^2 = 3x - 1$ **16.** $3x^2 = 2(2x + 1)$ **17.** $x(x - 5) = -4$

Solve each equation by factoring. Check your answers. See Problem 1.

6. $x^2 + 6x + 8 = 0$

To start, factor the quadratic expression. $(x + 2)(x + 4) = 0$

7. $x^2 + 18 = 9x$ **8.** $2x^2 - x = 3$ **9.** $x^2 - 10x + 25 = 0$

10. $2x^2 + 6x = -4$ **11.** $x^2 - 4x = 0$ **12.** $6x^2 + 4x = 0$

Simplifying Radicals

A radical expression is in simplest form when all of the following are true.

- The radicand has no perfect square factors other than 1.
- The radicand does not contain a fraction.
- A denominator does not contain a radical expression.

radical symbol

$\downarrow$

$\sqrt{a} \leftarrow$ radicand

Example 1

Simplify the expressions $\sqrt{2} \cdot \sqrt{8}$ and $\sqrt{294} \div \sqrt{3}$.

$\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$	Write both numbers under one radical.
$= \sqrt{16}$	Simplify the expression under the radical.
$= 4$	Factor out perfect squares and simplify.

$$\sqrt{294} \div \sqrt{3} = \sqrt{\frac{294}{3}} = \sqrt{98} = \sqrt{49 \cdot 2} = 7\sqrt{2}$$

Example 2

Write $\sqrt{\frac{4}{3}}$ in simplest form.

$\sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}}$	Rewrite the single radical as a quotient.
$= \frac{2}{\sqrt{3}}$	Simplify the numerator.
$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$	Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$ (a form of 1) to remove the radical from the denominator. This is called *rationalizing the denominator*.
$= \frac{2\sqrt{3}}{3}$	

Some help on this here.

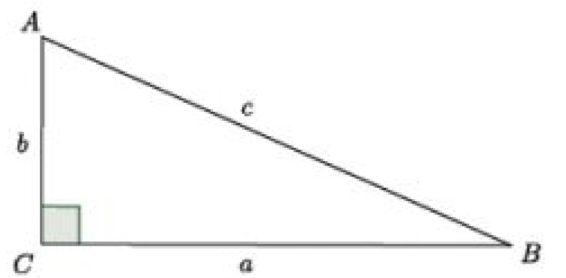
Exercises

Simplify each expression.

1. $\sqrt{5} \cdot \sqrt{10}$
2. $\sqrt{243}$
3. $\sqrt{128} \div \sqrt{2}$
4. $\sqrt{\frac{125}{4}}$
5. $\sqrt{6} \cdot \sqrt{8}$
6. $\frac{\sqrt{36}}{\sqrt{3}}$
7. $\frac{\sqrt{144}}{\sqrt{2}}$
8. $\sqrt{3} \cdot \sqrt{12}$
9. $\sqrt{72} \div \sqrt{2}$
10. $\sqrt{169}$
11. $28 \div \sqrt{8}$
12. $\sqrt{300} \div \sqrt{5}$
13. $\sqrt{12} \cdot \sqrt{2}$
14. $\frac{\sqrt{6} \cdot \sqrt{3}}{\sqrt{9}}$
15. $\frac{\sqrt{3} \cdot \sqrt{15}}{\sqrt{2}}$

Pythagorean's Theorem and Converse

Given a right triangle ABC with C being the vertex of the right angle, then the sides $\overline{AC}$ and $\overline{BC}$ are called the *legs* of $\triangle ABC$, and $\overline{AB}$ is called the *hypotenuse* of $\triangle ABC$.



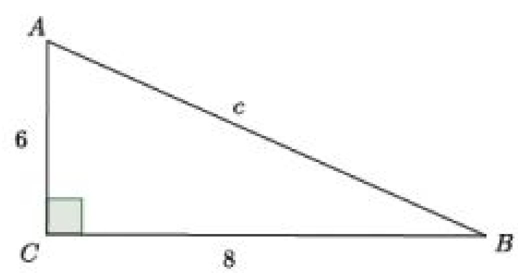
Take note of the fact that side a is opposite the angle A, side b is opposite the angle B, and side c is opposite the angle C.

The Pythagorean theorem states that for any right triangle, $a^2 + b^2 = c^2$.

Example 1

Now that we know what the Pythagorean theorem is, let's practice using it to find the length of a hypotenuse of a right triangle.

Determine the length of the hypotenuse of the right triangle.



The Pythagorean theorem states that for right triangles $a^2 + b^2 = c^2$, where a and b are the legs, and c is the hypotenuse. Then,

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2. \end{aligned}$$

Since we know that $100 = 10^2$, we can say that the hypotenuse c is 10.

The converse of the Pythagorean theorem states that if a triangle with side lengths a, b, and c satisfies $a^2 + b^2 = c^2$, then the triangle is a right triangle.

For more help on Pythagorean and Converse, click here.

Exercises

Determine whether the given lengths can be side lengths of a right triangle.

1. 15, 36, 39 **2.** 3, 7, 10 **3.** 8, 15, 17

4. $\sqrt{3}, \sqrt{4}, \sqrt{5}$ **5.** 6, 7, 8 **6.** 12, 16, 20

For the values given, *a* and *b* are legs of a right triangle. Find the length of the hypotenuse. If necessary, round to the nearest tenth.

7. $a = 6$, $b = 8$ **8.** $a = 5$, $b = 9$ **9.** $a = 4$, $b = 10$

10. $a = 9$, $b = 1$ **11.** $a = 7$, $b = 3.5$ **12.** $a = 1.4$, $b = 2.3$

Use the Pythagorean Theorem to answer each question.

13. A 20-ft ladder is placed 5 ft from the base of a building. How high on the building will the ladder reach?

14. A soccer field is 80 yd long and 35 yd wide. What is the diagonal distance across the field?

Basic Area

The formula to calculate the area of a parallelogram is $A = bh$, where b represents the base and h represents the height of the parallelogram.

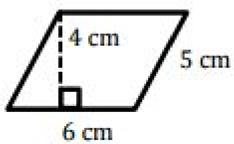
The height of a parallelogram is the line segment perpendicular to the base. The height is usually drawn from a vertex that is opposite the base.

Need help with area of <u>parallelograms</u> and <u>triangles</u>?

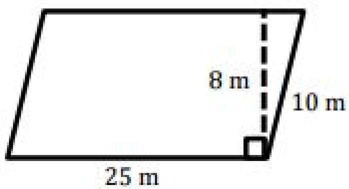
Exercises

1. Find the area of each parallelogram below. Note that the figures are not drawn to scale.

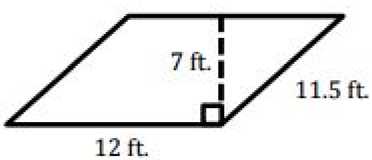
a.



b.



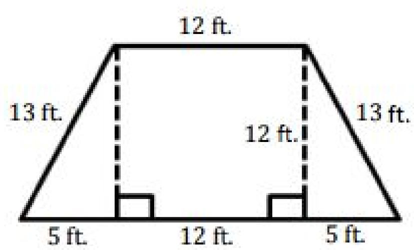
c.



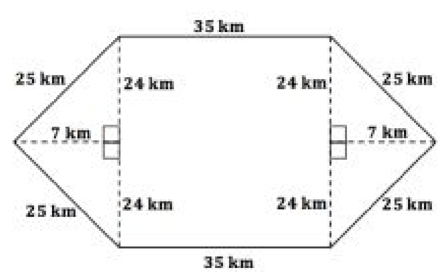
2. Can we use the formula $A = \frac{1}{2} \times \text{base} \times \text{height}$ to calculate the area of triangles that are not right triangles? Explain your thinking.

Calculate the area of each shape below. Figures are not drawn to scale.

3.

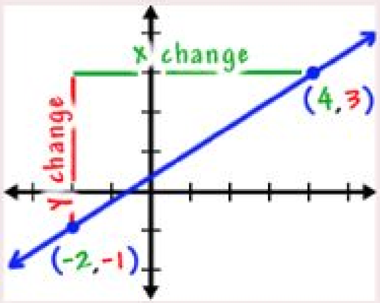


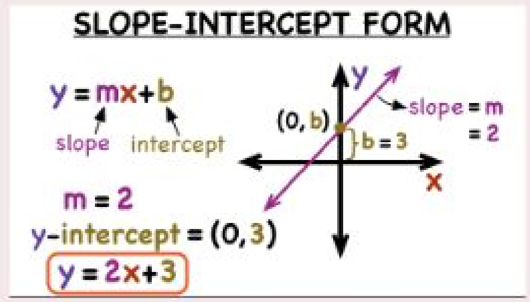
4.



5. Immanuel is building a fence to make an enclosed play area for his dog. The enclosed area will be in the shape of a triangle with a base of 48 m. and an altitude of 32 m. How much space does the dog have to play?

Slope

<table>
<tr><th>Slope</th><th>Slope-Intercept Form</th></tr>
<tr><td>A measure of the steepness of a line. If (x_1, y_1) and (x_2, y_2) are any two points on the line, the slope of the lines, known as m, is represented by the equation $\frac{y_2 - y_1}{x_2 - x_1}$.</td><td>The slope-intercept form for a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.</td></tr>
<tr><td>

Rise
Run</td><td>
</td></tr>
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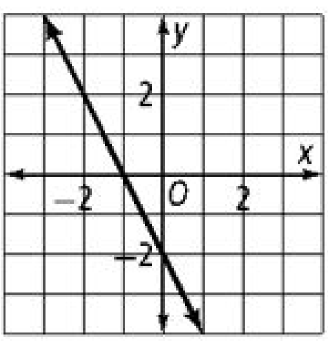
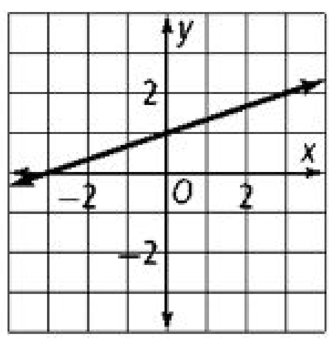
Some help on slope and equations of lines can be found here.

Find the slope of each line.

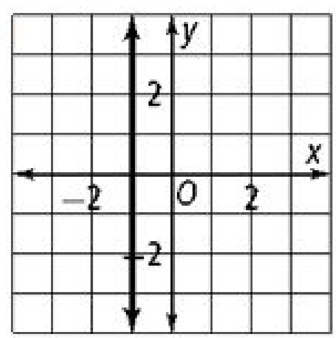
1.



2.



3.



Find the slope and *y*-intercept.

4. $y = 6x + 8$

5. $3x + 4y = -24$

6. $2y = 8$

7. $y = \frac{-3}{4}x - 8$

8. $2y = 3x - 1$

9. $4x - 5y = 2$

A line passes through the given points. Write an equation for the line in slope-intercept form.

10. (–2, 4) and (3, 9)

11. (1, 6) and (9, –4)

12. (0, –7) and (–1, 0)

13. (7, 0) and (3, –4)

14. (0, 0) and (–7, 1)

15. (10, 0) and (0, 7)